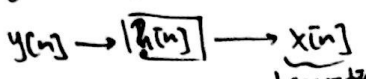
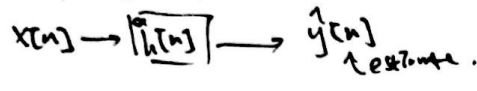


$y[n]$  is process



We want to derive  $\hat{x}[n]$  so that we can reconstruct the signal  $y[n]$  from observations.



Minimize error:

$e[n] = \hat{y}[n] - y[n] \Rightarrow \min_{h[n]} \epsilon = E\{e^2[n]\}$

Noncausal DT Wiener Filter:

$\epsilon = E\{e^2[n]\}$

$= E\left\{ \left( \sum_{k=-\infty}^{\infty} h[k]x[n-k] - y[n] \right)^2 \right\}$

*h here actually is the impulse response we are trying to estimate to get back y[n]*

Want to minimize  $\epsilon$ , so:

$\frac{\partial \epsilon}{\partial h[m]} = 0$  for all values of  $m$ , where  $h[n]$  is not restricted to be 0.

$\frac{\partial \epsilon}{\partial h[m]} = E\left\{ 2 \left( \sum_k h[k]x[n-k] - y[n] \right) x[n-m] \right\} = 0$

So,  $E\{2e[n]x[n-m]\} = 0$ , or  $R_{ex}[m] = 0 \Rightarrow e[n]$  and  $x[n]$  are orthogonal.

cross-correlation.

For the optimal filter, the error should be orthogonal to all the data used to construct the estimate.

Also,  
 $R_{ex}[m] = E\{e[n]x[n-m]\}$   
 $= E\{(\hat{y}[n] - y[n])x[n-m]\}$   
 $= R_{\hat{y}x}[m] - R_{yx}[m] = 0$

$\Rightarrow$  Optimal estimate would make it so  $R_{\hat{y}x}[m] = R_{yx}[m]$  for all appropriate  $m$ .

To derive the actual  $h[n]$ .. we have:

$R_{\hat{y}x}[m] = h[m] * R_{xx}[m]$   
 $R_{yx}[m] = h[m] * R_{xx}[m]$

$\Rightarrow \sum_k h[k]R_{xx}[m-k] = R_{yx}[m] \Rightarrow$  This is a set of linear equations we can solve for.

Suppose  $h[n]$  is restricted to  $n \in [0, N-1]$ , then:

$\begin{bmatrix} R_{xx}[0] & R_{xx}[-1] & \dots & R_{xx}[1-N] \\ R_{xx}[1] & R_{xx}[0] & \dots & R_{xx}[2-N] \\ \vdots & \vdots & \ddots & \vdots \\ R_{xx}[N-1] & R_{xx}[N-2] & \dots & R_{xx}[0] \end{bmatrix} \begin{bmatrix} h[0] \\ h[1] \\ \vdots \\ h[N-1] \end{bmatrix} = \begin{bmatrix} R_{yx}[0] \\ R_{yx}[1] \\ \vdots \\ R_{yx}[N-1] \end{bmatrix}$

This is the case when  $h[n]$  is an FIR, for example.

When  $h[n]$  is an IIR, we have infinite terms to solve!

But that means eq. (1) holds for  $m \in (-\infty, \infty)$ .

Use z-transform:

$H(z)S_{xx}(z) = S_{yx}(z)$

$H(z) = \frac{S_{yx}(z)}{S_{xx}(z)} \Rightarrow$  z-transform of transfer function can be found

If the Input/Output process does not have a z-transf., can use F.T domain calculations.

\*Caution: Going to frequency domain decouples the system.

$R_{\hat{y}x}[m] = R_{yy}[m] - R_{y\hat{y}}[m] = R_{yy}[m] - h[m] * R_{yx}[m]$

The last step comes from:

$R_{y\hat{y}} = E\left\{ y[n] \int \hat{y}[n-k] x[n-k] dk \right\} = R \{ h[m] * R_{yx}[m] \}$   
 $= E\left\{ \int y[n-k] h[k] x[n-k] dk \right\}$

The MMSE =  $R_{\hat{y}x}[0]$